Stability Regions of a Model Reference Control System

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Introduction

INSTABILITIES in linear systems can be generated by changing the parameters of the system periodically. This phenomenon is called parametric excitation. Perhaps the best known example is the Mathieu equation (see McLachlan[1]). Parametric excitation can also occur in adaptive systems. Even the simplest cases are very hard to analyze analytically. Attempts to compute the stability regions have been made earlier (see James[2]). These results have been used as examples in, e.g., Mareels[3] and Åström.[4] However, the results presented in James' paper are not completely accurate, as a new investigation has shown.

Problem and Solution

Consider the adjustment of a feedforward gain of a system with the transfer function $G(s) = 1/(s+1)$, using the MIT rule. The parameter adjustment rule is

$$\frac{d\theta}{dt} = -\gamma y_m(t) e(t) \tag{1}$$

where

$$e(t) = y(t) - y_m(t) \tag{2}$$

The system output is given by

$$\frac{dy}{dt} = \theta(t) u_c(t) - y(t) \tag{3}$$

where u_c is the reference signal, and y_m is the output from the desired model. The adaptive system is described in Fig. 1. Thus, both u_c and y_m can be regarded as known time-varying signals. The adaptive system is a time-varying dynamic system with two state variables θ and y. The system can be described by

$$\begin{pmatrix}\dot\theta\\ \dot y\end{pmatrix} = \begin{pmatrix}0 & -\gamma y_m(t)\\ u_c(t) & -1\end{pmatrix}\begin{pmatrix}\theta\\ y\end{pmatrix} + \begin{pmatrix}\gamma y_m^2(t)\\ 0\end{pmatrix}$$
$$= A(t)\begin{pmatrix}\theta\\ y\end{pmatrix} + b \tag{4}$$

If the command signal is $u_c(t) = \sin\omega t$, then

$$y_m = \frac{1}{\sqrt{1+\omega^2}} \sin(\omega t - \arctan\omega) \tag{5}$$

and Eq. (4) becomes a linear system with periodic coefficients.

The stability of a linear system with periodic coefficients can be analyzed with the aid of the following theorem.

Theorem. All solutions of the periodic equation

$$\dot x(t) = A(t)x(t), \qquad A(t+T) = A(t) \tag{6}$$

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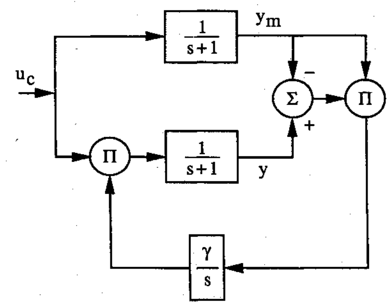

Fig. 1 Block diagram of the adaptive system.

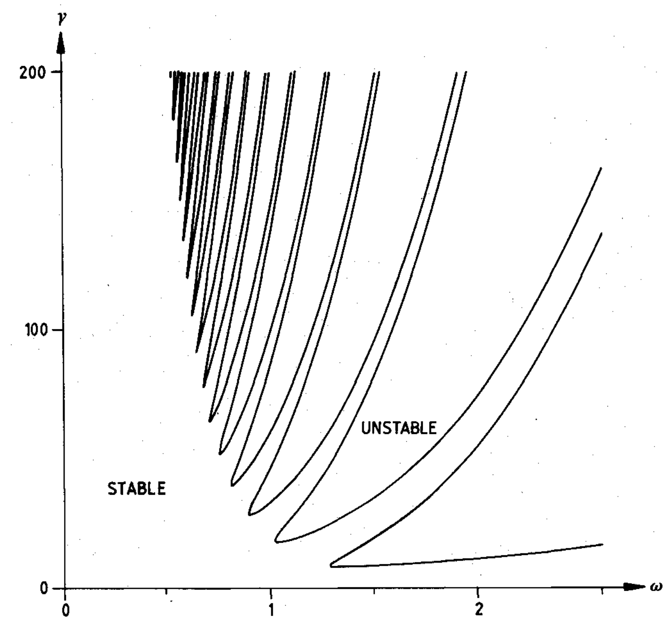

Fig. 2 Regions of stability.

approach zero as t approaches infinity if the zeros of $\det[Is - \Phi(t_0 + T, t_0)]$ lie in the disk $|s| < 1$.

The variable Φ is the transition matrix of the system $\dot x(t) = A(t)x(t)$. For proofs and more details, see Brocket.[5] The transition matrix is defined by the equations

$$\frac{d\Phi(t,t_0)}{dt} = A(t)\Phi(t,t_0), \qquad \Phi(t_0,t_0) = I \tag{7}$$

Solving Eq. (7) analytically is equivalent to solving

$$\ddot\phi + \left(1 - \omega\frac{\cos\omega t}{\sin\omega t}\right)\dot\phi + \left(\frac{\gamma \sin\omega t}{\sqrt{1+\omega^2}}\sin(\omega t - \arctan\omega) - \omega\frac{\cos\omega t}{\sin\omega t}\right)\phi = 0 \tag{8}$$

for which there is no method known to the author.

Equation (7) was integrated numerically with the $A(t)$ matrix defined in Eq. (4) to obtain $\Phi(T,0)$, and an equation of the second degree was solved to compute the eigenvalues of the transition matrix. The computations were carried out using the simulation language Simnon (see Elmqvist et al.[6]). The system was investigated for 500 ω values between 0.1 and 2.6 and for 500 γ values between 0 and 200; i.e., in all, 250,000 systems were investigated. The computations required about 90 CPU hours on a VAX 11/780. The regions of stability are shown in Fig. 2.

This investigation shows that even a very simple adaptive system can have very complicated behavior. A very probable reason for the erroneous results in James[2] is a too sparse discretization of the γ-ω plane.

Conclusions

In this Note we have investigated a first-order system with unknown gain controlled by an adaptive controller using the MIT rule. The stability of the system has been investigated by computing the eigenvalues of the transition matrix of the system. The stability of the system depends on a gain in the controller and the frequency of the sinusoidal input to the system. The regions of stability exhibit very complicated behavior.

Acknowledgment

Professor K. J. Åström pointed out that there might be an error in James' article and initiated this investigation.

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Low-Authority Eigenvalue Placement for Second-Order Structural Systems

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Introduction

DURING the past two decades numerous researchers have directed their attention to developing methods for control of very large, highly flexible space structures. Of special interest is precise attitude maneuvering and targeting of flexible satellite systems; thus, an extremely important problem for the dynamics and control community is that of suppressing the vibratory motion of a flexible structure due to external disturbances and/or active attitude maneuvers.[1] Eigenvalue assignment techniques using linear output feedback have been developed for multi-input/multi-output systems characterized by both the ($2n$) first-order state space form[2-5] and the (n) second-order structural configuration-space form.[6-8] In this Note, the concepts of low-authority control (i.e., 10–20% damping) and linear feedback control are combined with optimal linear programming to provide a systematic approach to optimize symmetric displacement and velocity feedback gain matrices utilizing the properties of second-order structural systems. Especially important is the development of a general algorithm to accommodate directly both eigenvalue placement and actuator saturation constraints.

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Structural Feedback Controller Design

The discretized equations of motion for a second-order structural system can be written in the standard form

$$M\ddot{x} + Kx = f_c + f_e \tag{1}$$

where M is the ($n \times n$) symmetric, positive-definite mass matrix, K is the ($n \times n$) symmetric, positive-semidefinite stiffness matrix, x is the ($n \times 1$) generalized coordinate vector, f_c is the ($n \times 1$) generalized control force vector, f_e is the ($n \times 1$) generalized external (disturbance) force vector, and the overdot represents differentiation with respect to time. The unforced version of Eq. (1) yields the open-loop system natural frequencies (ω_j) and mode shapes (ϕ_j). If a control law is chosen so that the closed-loop system has the form of a viscous-damped structure with collocated linear actuators and sensors, the control force vector and measurement vector take on the following form:

$$f_c = D^T u = -D^T H y_1 - D^T G y_2 \tag{2a}$$

$$y_1 = D\dot{x} \tag{2b}$$

$$y_2 = Dx \tag{2c}$$

where D^T is the ($n \times m$) control influence matrix, u is the ($m \times 1$) control vector, G and H are ($m \times m$) fully populated, symmetric positive-definite gain matrices, and y_1 and y_2 are ($m \times 1$) measurement vectors. The closed-loop equations can now be written as

$$M\ddot{x} + D^T H D\dot{x} + (K + D^T G D)x = f_e \tag{3}$$

By choosing the Lyapunov function $2V = \dot{x}^T M \dot{x} + x^T \times (K + D^T G D)x$, it can easily be shown that the system described by Eq. (3) will be stable (for bounded f_e) as long as G and H are symmetrical positive-definite matrices.[7] In fact, this system will remain stable even in the presence of large, unspecified (linear elastic) modeling errors, as long as the feedback law is constrained to satisfy the aforementioned definiteness properties. In the sequel a design approach is introduced that guarantees the desired properties of the gain matrices.

It is convenient to rewrite Eq. (1) in the symmetrical state-space form

$$\begin{bmatrix} -K & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} + \begin{bmatrix} 0 & K \\ K & 0 \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} = \begin{Bmatrix} 0 \\ f_c + f_e \end{Bmatrix} \tag{4}$$

which, upon implementing the control force of Eq. (2) and solving for the corresponding eigensolution, yields a set of eigenvector orthonormality conditions of the form

$$\{\Phi_j^T \;\; \lambda_j \Phi_j^T\} \begin{bmatrix} -K - D^T G D & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \Phi_j \\ \lambda_j \Phi_j \end{Bmatrix} = 1 \tag{5a}$$

$$\{\Phi_j^T \;\; \lambda_j \Phi_j^T\} \begin{bmatrix} 0 & K + D^T G D \\ K + D^T G D & D^T H D \end{bmatrix} \begin{Bmatrix} \Phi_j \\ \lambda_j \Phi_j \end{Bmatrix} = -\lambda_j \tag{5b}$$

where λ_j and Φ_j are the jth closed-loop eigenvalues and eigenvectors, respectively. Motivated by Creamer,[10] an approximation to Eqs. (5) can be written in the form

$$\alpha_j^2 \{\phi_j^T \;\; \lambda_j^* \phi_j^T\} \begin{bmatrix} -K - D^T G D & 0 \\ 0 & M \end{bmatrix} \begin{Bmatrix} \phi_j \\ \lambda_j^* \phi_j \end{Bmatrix} = 1 \tag{6a}$$

$$\alpha_j^2 \{\phi_j^T \;\; \lambda_j^* \phi_j^T\} \begin{bmatrix} 0 & K + D^T G D \\ K + D^T G D & D^T H D \end{bmatrix} \times \begin{Bmatrix} \phi_j \\ \lambda_j^* \phi_j \end{Bmatrix} = -\lambda_j^* \tag{6b}$$